

Q1 Key Exchange Protocols

(3 points)

Recall that in a Diffie-Hellman key exchange, there are values a, b, g , and p . Alice computes $g^a \bmod p$ and Bob computes $g^b \bmod p$.

Q1.1 (1 point) Which of these values (a, b, g , and p) are publicly known and which must be kept private?

a	b	g	p
☐ Public	☐ Public	☐ Public	☐ Public
☐ Private	☐ Private	☐ Private	☐ Private

Q1.2 (1 point) Mallory can eavesdrop, intercept, and modify everything sent between Alice and Bob. Alice and Bob perform Diffie-Hellman to agree on a shared symmetric key K . After the exchange, Bob gets the feeling that something went wrong and calls Alice. He compares his value of K to Alice's and realizes that they are different. Explain what Mallory has done.

Q1.3 (1 point) Assume that K , the Diffie-Hellman exponents a and b , and the messages themselves are destroyed once all messages are sent. That is, these values are not stored on Alice and Bob's devices after they are done communicating.

Eavesdropper Eve records all communications between Alice and Bob, but is unable to decrypt them. At some point in the future, Eve is lucky and manages to compromise Bob's computer.

Is the confidentiality of Alice and Bob's prior **Diffie-Hellman**-based communication in jeopardy? Explain why.

☐ Yes ☐ No



Q2 *Ra's al Gamal*

(3 points)

Recall the ElGamal scheme from lecture:

- $\text{KeyGen}() = (b, B = g^b \bmod p)$
- $\text{Enc}(B, M) = (C_1 = g^r \bmod p, C_2 = B^r \times M \bmod p)$

Q2.1 (1 point) Is the ciphertext (C_1, C_2) decryptable by someone who knows the private key b ? If you answer yes, provide a decryption formula. You may use C_1 , C_2 , b , and any public values.

☐ Yes

☐ No

Q2.2 (1 point) Consider an adversary that can efficiently break the discrete log problem. Can the adversary decrypt the ciphertext (C_1, C_2) without knowledge of the private key? If you answer yes, briefly state how the adversary can decrypt the ciphertext.

☐ Yes

☐ No

Q2.3 (1 point) Consider an adversary that can efficiently break the Diffie-Hellman problem. Can the adversary decrypt the ciphertext (C_1, C_2) without knowledge of the private key? If you answer yes, briefly state how the adversary can decrypt the ciphertext.

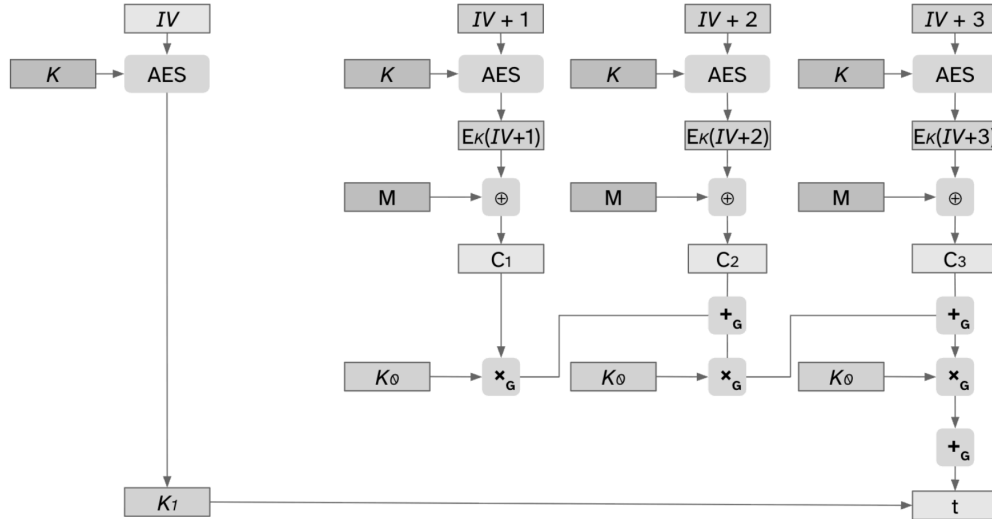
☐ Yes

☐ No

Q3 AES-GCM

(6 points)

Consider the **AES-GCM** tag-generation mechanism where plaintext blocks are encrypted under AES-CTR to produce ciphertext $C_1, C_2, \dots, C_n$. Then, the tag t is computed by combining these ciphertext blocks with the sub-keys $K_0 = H = E_K(0)$ and $K_1 = E_K(IV)$.



Assume there is no associated data, and assume the length block is omitted from the GHASH computation. So for an n -block ciphertext:

$$t = C_1 K_0^n \oplus C_2 K_0^{n-1} \oplus \dots \oplus C_n K_0 \oplus K_1$$

Q3.1 (1 point) In AES-GCM, what is the value of t for a 3-block message (M_1, M_2, M_3) ?

- ☐ $t = C_3 K_0^3 \oplus C_2 K_0^2 \oplus C_1 K_0 \oplus K_1$
- ☐ $t = C_1 K_0^3 \oplus C_2 K_0^2 \oplus C_3 K_0 \oplus K_1$
- ☐ $t = C_1 K_1^3 \oplus C_2 K_1^2 \oplus C_3 K_1 \oplus K_0$
- ☐ $t = C_3 K_1^3 \oplus C_2 K_1^2 \oplus C_1 K_1 \oplus K_0$

(Question 3 continued...)

Q3.2 (5 points) Assume you have access to an encryption oracle which takes your message M and outputs $M \oplus E_K(IV + 1)$. Now suppose the IV is reused across every ciphertext in this question, so K_0 and K_1 are the same fixed (unknown) values throughout. You know that $C = (C_1, C_2, C_3)$ has tag t . K is unknown.

What should the first ciphertext be?

The MAC of the message in the box above is t_a .

What should the second ciphertext be?

The MAC of the message in the box above is t_b .

Now, construct a three-block message C' with the same tag as C (i.e. with the tag t):

$C_{\text{new}} = \left(\begin{array}{|c|} \hline \text{First block} \\ \hline \end{array}, \begin{array}{|c|} \hline \text{Second block} \\ \hline \end{array}, \begin{array}{|c|} \hline \text{Third block} \\ \hline \end{array} \right)$